

Cosmic D-Strings and Vortons in Supergravity

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August 15, 2006

Abstract

Recent developments in string inspired models of inflation suggest that D-strings are formed at the end of inflation. Within the supergravity model of D-strings there are $2(n-1)$ chiral fermion zero modes for a D-string of winding n . Using the bounds on the relic vorton density, we show that D-strings with winding number $n > 1$ are more strongly constrained than cosmic strings arising in cosmological phase transitions. The D-string tension of such vortons, if they survive until the present, has to satisfy $8\pi G_N \mu \lesssim p 10^{-26}$ where p is the intercommutation probability. Similarly, D-strings coupled with spectator fermions carry currents and also need to respect the above bound. D-strings with $n = 1$ do not carry currents and evade the bound. We discuss the coupling of D-strings to supersymmetry breaking. When a single $U(1)$ gauge group is present, we show that there is an incompatibility between spontaneous supersymmetry breaking and cosmic D-strings. We propose an alternative mechanism for supersymmetry breaking, which includes an additional $U(1)$, and might alleviate the problem. We conjecture what effect this would have on the fermion zero modes.

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1 Introduction

The formation of cosmic strings appears to be a generic feature of recent models of brane inflation arising from fundamental string theory [1, 2]. Indeed, lower dimensional branes are formed when a brane and anti-brane annihilate with the production of $D3$ and $D1$ branes, or D-strings, favoured [3]. It has been argued that D-strings have much in common with cosmic strings in supergravity theories and that they could be identified with D-term strings [4]. Whilst cosmic strings in global supersymmetric theories have been analysed before [5, 6], the study of such strings with local SUSY is at an early stage since the presence of supergravity produces added complications. These have recently been addressed in refs. [4, 7, 8]. In ref. [8] an exhaustive study of fermion zero modes was performed. It was found that due to the presence of the gravitino, the number of zero mode solutions in supergravity is reduced in some cases. In particular, it was found that there are no zero modes for F-term strings. For BPS D-strings with winding number n the number of chiral zero modes is $2(n-1)$, rather than $2n$ in the global case. For winding number 1 strings this was observed in ref. [9]. When there are spectator fermions present, as might be expected if the underlying theory gives rise to the standard model of particle physics, then there are n chiral zero modes per spectator fermion.

Ordinary cosmic strings are either long strings or loops [10]. When two strings meet they intercommute with unit probability. When a long string self-intersects it forms a long string and a loop. String loops decay via gravitational radiation. This results in the string network rapidly reaching a scaling solution, and the string loops never dominate the energy density of the universe. This picture is changed in the presence of fermion zero modes. The zero modes move along the string resulting in the string carrying a current [11]. Loops can be stabilised by the presence of the current-carriers, giving rise to vortons [12]. These can be used to constrain the underlying theory [13, 14]. For cosmic strings or D-strings arising from fundamental string theory the intercommutation probability is less than unity [15]. This results in a slower, denser string network with a higher string loop density [16, 17]. If such strings are current-carrying then the resulting vorton density could be higher. We use bounds from both nucleosynthesis, and the closure of the universe, to constrain the D-string tension for vortons surviving until either Big Bang Nucleosynthesis, or the present time. Such bounds are extremely stringent. Of course the $n = 1$ winding D-strings evade these bounds, unless they couple to spectator fermions. Finally we discuss the embedding of the D-strings in a broken SUSY environment. This is of great phenomenological importance. We find obstacles to embedding D-strings in broken SUSY with a single $U(1)$ gauge group. We discuss a way round this by introducing an extra $U(1)$, and conjecture what effect this will have on the fermion zero modes.

The letter is arranged as follows. In section 2 we describe the properties of D-strings in supergravity and count the number of zero modes. In section 3, we analyse the constraints on the D-string tension coming from the relic density of vortons during BBN and today. Finally in section 4 we present some results on the coupling between D-strings and SUSY breaking.

2 D-strings and their Fermion Zero Modes

In this section we use the supergravity description of D-strings by D-term strings [4]. Consider a supergravity theory with fields ϕ^\pm , charged under an Abelian gauge group. The D-term bosonic potential

$$V = \frac{g^2}{2}(|\phi^+|^2 - |\phi^-|^2 - \xi)^2 \quad (2.1)$$

includes a non-trivial Fayet-Iliopoulos term ξ . Such a term is compatible with supergravity provided the superpotential has charge $-\xi$. Here the superpotential vanishes. The minimum of the potential is $|\phi^+|^2 = \xi$. It is consistent to take the cosmic string solution to be

$$\phi^+ = f(r)e^{in\theta} \quad (2.2)$$

where n is the winding number, and $\phi^- = 0$ since this minimises the corresponding vector mass [18]. The function $f(r)$ interpolates between 0 and $\sqrt{\xi}$. The presence of a cosmic string bends space-time. Its gravitational effects lead to a deficit angle in the far away metric of spacetime. In the following we consider the metric

$$ds^2 = e^{2B}(-dt^2 + dz^2) + dr^2 + C^2 d\theta^2 \quad (2.3)$$

for a cosmic string configuration. This is the most general cylindrically symmetric metric, as discussed by Thorne [19]. Far away from the string the energy momentum tensor is approximately zero and therefore B is zero. Now we also find that

$$C = C_1 r + C_0 + O(r^{-1}) , \quad (2.4)$$

When $C_1 \neq 0$, the solution is a cosmic string solution with a deficit angle $\Delta = 2\pi(1 - C_1)$.

In supersymmetric theories, a cosmic string generally breaks all supersymmetries in its core. BPS objects are an exception to this rule, as they leave 1/2 of the original supersymmetry unbroken. D-strings, in which we will be interested in this paper, are an example of this. These strings have vanishing T_{rr} , and the conformal factor B is identically zero. Moreover for D-strings one finds

$$\Delta = 2\pi|n|\xi . \quad (2.5)$$

The supersymmetry algebra in four dimensions allows for 1/2 BPS configurations which saturate a BPS bound giving an equality between the mass, i.e. the tension, and a central charge. Other cosmic strings have higher tension and are not BPS, i.e. they break all supersymmetries. This implies that C_1 is less than the BPS case giving

$$\Delta \geq 2\pi|n|\xi . \quad (2.6)$$

Let us now consider the fermion partners of the D-string bosonic fields. These are the higgsino χ^+ , the gaugino λ , and gravitino ψ_μ . On D-strings, the number of zero mode

bound states is drastically affected by supergravity effects. We will discuss D-term strings and then add spectator fields as might be the case when embedding D-strings in a particle physics model.

Let us first consider the model in global supersymmetry [5]. In this case the zero modes can be constructed explicitly. The zero mode solutions have definite chirality, in the sense that they are eigenstates of σ^3 . We find that for large r , there is one solution for which λ and χ decay exponentially. For positive chirality zero modes, we find that this solution has the form

$$\lambda \sim r^m e^{i(m+1/2)\theta}, \quad \chi^+ \sim r^{n-1-m} e^{i(n-m-1/2)\theta}, \quad (2.7)$$

near the origin. For the solution to be normalisable using the \mathcal{L}^2 norm, the integer m must satisfy $0 \leq m \leq n-1$. Denoting the number of normalisable zero mode solutions of chirality \pm by N^\pm , we find

$$N^+ = 2n, \quad N^- = 0 \quad (2.8)$$

for $n > 0$. For negative n the two chiralities are interchanged, and there are $2|n|$ zero modes with negative chirality.

In contrast to the global case, we must now include the gravitino field. In supergravity, zero modes are obtained once a gauge choice has been made for the supersymmetry transformations. It is particularly convenient to work in the $\bar{\sigma}^\mu \psi_\mu = 0$ gauge. In this gauge, the gravitino has three degrees of freedom, each being a Weyl fermion. The equations of motion for the gravitino in a BPS background imply that $\psi_t = \psi_z = 0$. The remaining degree of freedom is a single Weyl fermion, $\Psi = \sigma^r \bar{\psi}_r - \sigma^\theta \bar{\psi}_\theta$. It has the same norm as the other non-gravitino Weyl fermions. Moreover the field Ψ has the same chirality as the ordinary fermions.

Near $r = 0$ the general solution to the fermion field equations, for positive chirality states, has the leading order behaviour

$$|\chi^+| = c_1 r^{n-m-1}, \quad |\lambda| = c_2 r^m, \quad |\Psi| = c_3 r^{m-1}, \quad (2.9)$$

where the c_i are constants. At infinity just one combination of solutions decays exponentially, and (for $m \geq 0$) is the only normalisable solution there. In general, the form of this combination of solutions near $r = 0$ will be given by the above expression with all $c_i \neq 0$. Hence if the solution is to be normalisable everywhere, we must have $m \leq n-1$, $m \geq 0$ and $m \geq 1$ (the last condition comes from the gravitino field Ψ). Without the gravitino we would need $n-1 \geq m \geq 0$. Including it we lose the $m = 0$ mode as it cannot be normalisable close to the origin. The rest of the zero mode tower is preserved. Notice that this effect is purely local and does not depend on the behaviour of the fields at infinity. For $n > 0$ the number of zero modes is therefore

$$N^+ = 2(n-1), \quad N^- = 0. \quad (2.10)$$

This can be confirmed using the generalised index theorem [8, 20].

The D-string model can be augmented with new spectator fields. These fields may appear in particle physics models. We add to the D-strings new fields Φ_i and a superpotential

$$W = \sum_{i=1}^M \frac{a_i}{2} \phi^+ \Phi_i^2 \quad (2.11)$$

These fields are massive outside the string core. In the string background, we have $\Phi_i = 0$. The fermion χ_i associated with Φ_i does not then couple to the string sector, and so it can be analysed independently. Applying the index theorem (with $n > 0$) we find

$$N^+ = 0, \quad N^- = nM. \quad (2.12)$$

The presence of these chiral zero modes has been obtained in ref. [7]. The above result also holds for global SUSY, and is not affected by the inclusion of SUGRA effects. The result will be unaffected by supersymmetry breaking.

3 Cosmic Vortons

The existence of fermion zero modes in the string core can have dramatic consequences for the properties of cosmic strings. For example, the zero modes can be excited and will move along the string, resulting in the string carrying a current [11]. The direction of the charge carriers is determined by the chirality of the zero modes. If they all have the same chirality, they will all move in the same direction. The current will be maximal, and chiral. An initially weak current on a string loop is amplified as the loop contracts and could become sufficiently strong to prevent the string loop from decaying. In this case a stable loop, or vorton [12], is produced. Vortons are classically stable [21], so if they do decay, then its probably via quantum mechanical tunnelling, resulting in them being very long lived. In the case of fermion zero modes it has been shown that they are indeed very stable [22], particularly when the zero modes are chiral such as those under consideration here [23]. The density of vortons is tightly constrained by cosmology. In particular, if vortons are sufficiently stable so that they survive until the present time then the universe must not be vorton dominated. However, if vortons only survive a few minutes then they can still have cosmological implications since the universe must be radiation dominated at nucleosynthesis. These requirements have been used to constrain such models [13, 14]. In the case of strings arising from a fundamental string theory there is another consideration to take into account. In ref. [15] it was shown that for such strings the probability of intercommutation, p , is less than unity. For D-strings it was estimated that p is between $0.1 < p < 1$. Strings with lower p are slower and have a denser string network, resulting in the number of loops being proportional to p^{-1} [16, 17]. Consequently we need to recalculate the vorton bounds taking into account the reduced intercommutation probability. A string loop has two conserved quantum numbers; N is the topologically conserved integral of the phase current and Z is the particle number, which are identically equal for chiral currents. This results in the angular momentum quantum number $J = N^2$. The energy of the vorton

is $E_v \simeq \ell_v \mu$, where μ is the string tension. Taking the vorton to be approximately circular, the radius is $R_v = \ell_v/2\pi$, which gives $\ell_v \simeq (2\pi)^{1/2} N \mu^{-1/2}$. Thus we obtain an estimate of the vorton mass energy as

$$E_v \simeq N \mu^{1/2} , \quad (3.1)$$

where we are assuming the classical description of the string dynamics. This is valid only if the length ℓ_v is large compared with the relevant quantum wavelengths, which is satisfied if N is sufficiently large. A loop that does not satisfy this requirement will never stabilise as a vorton. We can now calculate the vorton abundance, following [14] but remembering that the intercommutation probability is less than unity. Assuming that the string becomes current carrying at a scale T_x then one expects that thermal fluctuations will give rise to a current. For strings arising from a cosmological phase transition, T_x is the transition temperature. Whilst for strings arising in brane inflation models T_x is the relevant string scale [24]. In both cases we expect $T_x \simeq \mu^{1/2}$. Loops with $N \gg 1$ should satisfy the minimum length requirement, otherwise the loop is doomed to lose all its energy and disappear. The total number density of small loops with length of order L_{\min} , the minimum length for vortons, will be not much less than the number density of all closed loops and hence $n \simeq L_{\min}^{-3}(T)p^{-1}$. The typical length scale of string loops at the transition temperature, $L_{\min}(T_x)$, is considerably greater than the relevant thermal correlation length, T_x^{-1} , resulting in a modified string loop evolution. Loops present at the time of the current condensation then satisfy $L \geq L_{\min}(T_x)$, and reasonably large values of the quantum number N build up. If λ is the wavelength of the fluctuation of the carrier field then $N \simeq L/\lambda$, where $\lambda \simeq T_x^{-1}$. Thus, one obtains

$$N \simeq L_{\min}(T_x)T_x \gg 1 . \quad (3.2)$$

For current condensation during the early friction-dominated regime of string network evolution this requirement is always satisfied. Therefore, the vorton mass density is $\rho_v \simeq N \mu^{1/2} n_v$. In the friction-dominated regime the string is interacting with the surrounding plasma. We can estimate L_{\min} in this regime as the typical length scale below which the microstructure is smoothed, with the dominant mechanism being Aharonov-Bohm scattering. This length scale is $(m_p \mu)^{1/2}/[(g^*)^{1/4} T^{5/2}]$, where $g^*(T)$ is the effective number of massless degrees of freedom for the plasma. This then gives the quantum number, $N \simeq m_p^{1/2}/(g_x^* \mu)^{1/4}$, giving the number density of mature vortons $n_v \simeq g^* T^3 \mu^{3/4}/[(g_x^*)^{1/4} m_p^{3/2} p]$. The resulting mass density of the relic vorton population is

$$\rho_v \simeq \left(\frac{\mu}{\sqrt{g_x^*} m_p} \right) \frac{g^* T^3}{p} , \quad (3.3)$$

We are now in a position to bound the energy scale of formation of current carrying strings.

The standard cosmological model requires the universe to be radiation dominated at the time of nucleosynthesis. Thus the energy density in vortons at that time, $\rho_v(T_N)$, should have been small compared with the background energy density in radiation, $\rho_N \simeq g^* T_N^4$. Assuming that carrier condensation occurs during the friction damping regime, this gives

$$\left(\frac{\mu}{p \sqrt{g_x^*} m_p} \right) \ll T_N , \quad (3.4)$$

where g_x^* is the effective number of degrees of freedom at the time of current condensation. If we make the rather conservative assumption that vortons only survive for a few minutes, which is all that is needed to reach the nucleosynthesis epoch, we obtain a fairly strong restriction on the theory:

$$8\pi G_N \mu \lesssim p \sqrt{g_x^*} 10^{-22} \quad (3.5)$$

where G_N is Newton's constant. Taking $g_x^* \simeq 10^2$ in the early universe we obtain

$$8\pi G_N \mu \lesssim p 10^{-21} . \quad (3.6)$$

If vortons are sufficiently stable to survive until the present epoch one can find a more stringent constraint by requiring that the vortons do not dominate the energy density today. We impose that $\Omega_v \equiv \rho_v/\rho_c \leq 1$, where ρ_c is the closure density. Inserting our earlier estimate for the vorton density, we can derive the dark matter constraint. This gives

$$8\pi G_N \mu \lesssim p \sqrt{g_x^*} 10^{-27} \simeq p 10^{-26} . \quad (3.7)$$

This result is based on the assumptions that the vortons are stable enough to survive until the present day. However, we expect it to be realistic for the chiral case since such vortons are classically and quantum mechanically very stable. These constraints put strong limits on D-strings with fermion zero modes. We note that $n = 1$ D-strings would evade these constraints, but higher winding number strings and D-strings with spectator fermions would be subject to the above limits. These limits are much stronger than estimates of the string tension for D-strings formed in brane inflation models.

4 Coupling D-strings to Spontaneously Broken Supersymmetry

In the previous section we have studied the properties of a vorton producing string network. In a realistic context, one must take into account that supersymmetry will be broken; coupling a SUSY-breaking sector to a D-string is non-trivial. The SUSY-breaking will change the microphysics of the string, and we also expect it to alter the conductivity of the strings and the corresponding vorton constraints.

In order for the Fayet-Iliopoulos term ξ to be present in the theory, the superpotential must have charge $-\xi$. It can therefore be written as

$$W = (\phi^+)^{-\xi} W_{SB}(\dots) \quad (4.1)$$

where the function W_{SB} depends on only uncharged combinations of fields, and so is uncharged itself. This means that in a string background the gravitino mass $m_{3/2} \propto W \propto e^{-in\xi\theta}$. For most string solutions this will not be single valued (there is no problem before SUSY-breaking since $W = 0$ then).

In general, when supersymmetry is broken, there will be a discontinuity in the phase of $m_{3/2}$ around the strings. Domain walls will form along this discontinuity. A string network

will therefore develop a network of domain walls between the different strings. The walls will try to contract, pulling the strings together until they either annihilate, or form higher winding number strings with single valued $m_{3/2}$ around them. Alternatively the walls will not contract quickly enough and we will be left with a network of domain walls, which will conflict with observation.

For the above model the gravitino mass will only be single valued if $n\xi$ is an integer, resulting in a supermassive string [10]. The string deficit angle is then $\Delta = 2\pi(1 - C_1) \geq 2\pi|n|\xi$. This implies, for $n \neq 0$, that the deficit angle is greater or equal to 2π . If $\Delta = 2\pi$ the space outside the string is topologically equivalent to $S^1 \times \mathcal{R}^2$, which is unphysical. On the other hand if $\Delta > 2\pi$ there is a curvature singularity at finite distance from the string where $C(r) = 0$, which is again unphysical. This seems to suggest that D-strings and SUSY-breaking cannot be combined in the same model.

A possible way round this problem would be to include an additional $U(1)$ in the theory, with a corresponding Fayet-Iliopoulos term $\tilde{\xi}$. We also include an additional field ρ which has unit charge under the new gauge group. For the superpotential to have the right gauge dependence it must now have the form

$$W = (\phi^+)^{-\xi} \rho^{-\tilde{\xi}} W_{SB}(\dots) . \quad (4.2)$$

We assume that after SUSY-breaking ρ gains a VEV, and we suppose that it winds around the string $\rho \propto e^{im\theta}$. This implies $m_{3/2} \propto e^{-i(n\xi + m\tilde{\xi})\theta}$. Hence if n , m and $n\xi + m\tilde{\xi}$ are all integers, the string will be single valued. If $\xi/\tilde{\xi}$ is rational it is always possible to find suitable n and m . It is energetically favourable for the winding m to settle down to a value for which all fields, and $m_{3/2}$, are single-valued (assuming such a value exists). Otherwise there will be a sharp jump in the phase of ρ at some value of θ . The energy of the resulting domain wall means that such a configuration is disfavoured. For some values of $\xi/\tilde{\xi}$, string solutions with $n = 1$ will not have single-valued $m_{3/2}$ for any choice of m . In this case walls will inevitably form between different strings. The domain wall tension will then pull the strings together until a configuration with a higher n , which does allow single-valued $m_{3/2}$, is produced. Alternatively the force between the strings may be too weak, and we will instead be left with a network of domain walls.

If W_{SB} and the coupling of the extra D-term are small, we expect their contribution to the string mass per unit length, and also its deficit angle to be small. Thus, assuming a self consistent model of the above form can be constructed, D-strings are compatible with SUSY-breaking. Hence the vorton constraints from section 3 will still apply. The above situation is similar to an $SO(10)$ string at electroweak symmetry breaking [25]. There it seems that the electroweak Higgs will not be single valued around the string. However, by taking into account the effect of a $U(1)$ subgroup of the electroweak sector, the problem is avoided.

Since the form of the string is changed by SUSY-breaking, it is possible that its conductivity, and the corresponding vorton bounds will also be altered. We see that SUSY-breaking couples more fermion fields to the string, and that the solutions also have more Yukawa couplings which wind around the string. From the index theorem [8, 20] we find

that both these factors tend to increase string conductivity. This means that the strings could become conducting after SUSY-breaking, even if they do not conduct before. The corresponding vorton bounds in this case are weaker, as discussed in refs. [13, 14]. On the other hand we expect that there will be some cases where the conductivity is reduced by SUSY-breaking.

Vorton constraints arising from spectator zero modes will not be affected by the above arguments. The spectator sector fermions do not couple to any of the string field fermions, so string conductivities in the two sectors are independent. Of course the spectator fermions could couple to other fields involved in SUSY-breaking, and this could alter the number of spectator zero modes.

5 Conclusions

In this letter we have seen that winding number n cosmic D-strings have $2(n - 1)$ chiral fermion zero modes in their core. Similarly cosmic D-strings with M spectator fermions have nM chiral zero modes in addition to those arising from the actual D-string itself. These zero modes can be excited and give rise to currents, which can stabilise string loops. We have investigated the resulting bounds on stable string loops, or vortons, taking into account the fact that if the string arises from a higher dimensional underlying theory, the intercommutation probability will be reduced. This leads to more stringent bounds than their field theory counterparts. For vortons which are absolutely stable we found that $8\pi G_N \mu \lesssim p 10^{-26}$ where p is the intercommutation probability. For vortons that survive only a few minutes, probably decaying via quantum mechanical tunnelling, we found $8\pi G_N \mu \lesssim p 10^{-21}$. However, as discussed previously vortons formed from cosmic D-strings are likely to be particularly stable, so the dark matter constraint is probably the most realistic in this case. Both bounds are much stronger than bounds on the string tension arising from structure formation, and the estimates from brane inflation models. The latter is model dependent, but estimates suggest that the D-string tension is between $10^{-12} \lesssim G_N \mu \lesssim 10^{-6}$ [1].

We have also investigated the effect of supersymmetry breaking on cosmic D-strings. For BPS strings there is an incompatibility between soft supersymmetry breaking, as might arise from gluino condensation, and cosmic D-strings. This arises because the gravitino is charged under the Fayet-Iliopoulos term resulting in superpotentials having charge $-\xi$. The requirement that the gravitino mass is single valued implies that ξ must be quantised, giving a deficit angle bigger than 2π , which is unphysical. This seems to be an underlying problem with cosmic D-strings. To avoid this we have proposed another way of breaking supersymmetry in D-string theories, which includes an additional $U(1)$ symmetry. In order to ascertain the effect of this on the fermion zero modes requires a detailed model, which is beyond the scope of this paper and will be left for future investigation. However, it is unlikely such a mechanism would destroy the zero modes and could even result in introducing extra zero modes at the scale of supersymmetry breaking. If this were to happen the vorton bounds would change to take into account the two scales along the lines

of ref. [14].

Acknowledgements

ACD is grateful for CEA Saclay for hospitality during some of this work. This work was supported in part by PPARC, the Netherlands Organisation for Scientific Research (NWO), the RTN European programme MRTN-CT-2004-503369 and the ESF Coslab programme.

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